Sensitivity of Unlined Canal Seepage to Hydraulic Properties of Polyacrylamide-Treated Soil

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High-molecular-weight, anionic, linear polyacrylamide (PAM) is being evaluated as a means of sealing unlined water delivery canals to reduce seepage losses. In this study, we investigated water seepage loss reduction by using a two-layer one-dimensional steady-state conceptual model. The objectives of this study were twofold: (i) to examine the effectiveness and implications of a PAM-treated thin layer at the bottom of the water delivery canal in reducing water seepage loss, and (ii) to investigate the sensitivity of seepage reduction to the hydraulic parameters of both the untreated and PAM-treated thin layer. We incorporated laboratory-measured saturated soil hydraulic conductivity results into the model to examine the seepage ratio of PAM-treated vs. untreated soils and the sensitivity of the seepage ratio. In cases where PAM applications and suspended sediment concentration (SSC) treatment were effective, the uncertainty in the soil hydraulic parameter (sorptive number) \( \alpha \) ratio of the PAM-treated soil layer over the original soil had a significant impact on the uncertainty in the seepage ratio. For cases where the seepage ratio was insensitive to PAM and SSC treatment, the \( \alpha \) ratio of the treated soil layer over the original soil layer was not significant. For a canal underlain by coarse-textured material, uncertainty in the \( \alpha \) ratio is propagated to the uncertainty in the seepage ratio and augmented more significantly, while for relatively fine-textured sand (reflected in the small \( \alpha \) values used in this study) the uncertainty propagation and augmentation are relatively gradual. For coarse-textured sand, small uncertainty in the \( \alpha \) ratio can lead to large uncertainty in the prediction of the seepage ratio.

Abbreviations: CV, coefficient of variation; PAM, polyacrylamide; SSC, suspended sediment concentration.

Water is especially important in the western United States, where prolonged drought can and does occur. The USGS (1990) has estimated that as much as 50% of the water flowing through unlined water delivery canals could be lost from seepage through the canal bottoms. High-molecular-weight, linear, anionic polyacrylamide (PAM) has been suggested as a means of sealing unlined water delivery canals to reduce seepage or infiltration losses. Depending on the concentration of PAM, the water chemistry, soil structure, etc., infiltration rates through treated soil can either increase or decrease. While increasing infiltration or seepage is often advantageous for supplying water to crops from furrows or recharging groundwater aquifers, decreasing seepage loss is desirable if the only goal is to transport water from one location to another. The impact of PAM treatment on the infiltrative ability is partly a function of the PAM concentration and the electrolyte concentration of the solution. For example, at low concentrations, PAM has been shown to increase water infiltration into soils by preserving soil structure and reducing soil dispersion and surface seals (Sojka et al., 1998; Sirjaebs et al., 2000). Increasing the electrolyte concentration of the solution can also increase infiltration rates assuming a constant PAM concentration (Ajwa and Trout, 2006, Yu et al., 2003). Polyacrylamide treatment also reduces soil erosion and sediment concentration in runoff water (Lentz and Sojka, 1994; Bjorneberg and Aase, 2000; Lentz and Bjorneberg, 2003; among many others).

Using PAM as a means to reduce infiltration for water conservation, however, is a relatively new but important application that only recently has been considered. One of the earlier known reports of this use was by Valliant (1999), who showed in a scaled-down trough that PAM and sediment together could reduce seepage by 60%. Valliant (2002) also examined the effectiveness of PAM treatment in an active water delivery canal by showing that water levels in nearby water wells were significantly lower than in untreated canal reaches. Because of the ongoing drought in the western United States and the results of these field studies, the U.S. Bureau of Reclamation (USBR) began evaluating the use of PAM as a means of sealing unlined water delivery canals.

As part of the USBR-sponsored studies, Young et al. (2009), Moran and Young (2007), and Moran (2007) sought to quantify the interactions of PAM and SSCs that reduced the saturated hydraulic conductivity \( K_s \) of three sandy-textured materials, and to better understand the mechanisms contributing to the reduction in \( K_s \). Three possible physical mechanisms that reduce seepage were considered in Young et al. (2009): (i) PAM–sediment flocculates physically plug large soil pores, especially in coarser grained sands; (ii) PAM itself becomes a...
A low-conductivity layer, similar to the thin-layer concept described by Lentz (2003); and (iii) PAM-treated water is more viscous than untreated water, thus reducing the infiltration rate. Testing was conducted using a constant-head method in soil columns. Suspended sediment was continuously added to a constant-head water reservoir, into which PAM was added and mixed with an agitator. The specific PAM polymer used for the experiments was TACK Dry (Precision Polymer Corp., Greeley, CO). It is an anionic, linear polymer with a molecular weight of approximately 18 Mg mol$^{-1}$.

Three different materials were chosen by Young et al. (2009) to test PAM efficiency, including (i) a natural coarse sand of mixed particle sizes that conforms to the ASTM C33 designation and was collected from Grand Junction, CO, (ii) an engineered washed silica (predominantly) fine-textured sand (obtained from a local home improvement store) used as a control (hereafter called #70 mesh sand), and (iii) a loamy sand soil also collected from Grand Junction, CO. Particle size distributions of each material were determined using the laser light scattering technique (Digisizer, Micromeritics, Norcross, GA); results are shown in Table 1. Suspended sediment was used as an experimental variable to more closely replicate conditions in operational canals. To maintain consistency between experiments, kaolinitic material (type Huber 80, J.M. Huber Corp., Macon, GA) was used for all experiments where suspended sediment was needed. This material has a median particle diameter of 2.5 μm, a specific gravity of 2.60, and a surface area of 16 m$^2$ g$^{-1}$. This same material was also used in recent PAM experiments (Deng et al., 2006). The PAM treatment levels were 0, 5.6, 11.2, 22.4, and 44.8 kg ha$^{-1}$, and the SSC levels were 0, 150, and 300 mg L$^{-1}$. All column experiments used a test solution of deionized water containing 0.005 mol L$^{-1}$ CaSO$_4$.

The results of Young et al. (2009) showed that treatment reduced $K_s$ from 40 to 98% in the coarse-textured sands, but reductions were much less in loamy sand (0–56%). Combining PAM and suspended sediment reduced $K_s$ eight to 11 times more than adding PAM without suspended sediment. Mechanisms that reduced $K_s$ included the plugging of larger soil pores near the soil surface and higher viscosity from the dissolved PAM. The former mechanism dominated when PAM treatment exceeded 5.6 kg ha$^{-1}$ and when the SSC was 150 mg L$^{-1}$ or higher. Significant $K_s$ reductions were also observed when tests were run using only filter material (i.e., column setups without soil), indicating that a thin soil seal composed of PAM flocculates could partially explain the observed $K_s$ reduction. Increases in PAM concentration without any suspended sediment had the ability to decrease $K_s$ in all materials tested. A portion of this reduction could be accounted for by viscous effects in the test solution, but these effects were found to be secondary to the PAM accumulation at the soil surface, which either clogged larger pores or otherwise formed a distinct layer separate from the soil.

Despite these efforts to understand the mechanism of PAM treatment in laboratory experiments, we are unaware of controlled studies that have quantified the effects of PAM treatment under field conditions. Thus, given the differences between field and laboratory experimental conditions, the relatively new usage of PAM as a canal sealant, and the overall difficulty in controlling field conditions that affect PAM hydration and reactivity (i.e., turbidity levels, water temperature, etc.), a need exists to develop an overall better understanding of when and to what extent PAM could be an effective sealant.

In other studies, Lentz (2007) recently showed that cross-linked PAM could be effective in some cases for reducing seepage losses in unlined canals, even for soils with textures ranging from loam to clay loam. Lentz and Freeborn (2007), in a mini-flume study using linear, anionic PAM, presented mixed seepage control results with respect to the interaction of PAM, SSC, and particle size. They concluded that physical interactions of the experimental factors influenced the results but the specific nature of these interactions was not well understood.

In the study reported here, we tried to fill one of the knowledge gaps and develop a simple conceptual framework to (i) evaluate the effects on water seepage of forming thin, PAM-treated layers, and (ii) examine the sensitivity of seepage reduction to the hydraulic parameters of both untreated and PAM-treated conditions. Our emphasis was focused especially on the sensitivity of the seepage ratio (i.e., the ratio of the seepage rate for the system involving PAM-treated soils divided by that for the untreated soil system) to the hydraulic parameter, $\alpha$, found in Gardner’s (1958) hydraulic conductivity equation for soils treated with PAM. The study incorporated $K_s$ results from the laboratory experiments of Young et al. (2009) and applied them to field scenarios using a conceptual framework for vertical flow in layered soil systems that would be applicable to canal environments.

### MATERIALS AND METHODS

#### Seepage Ratio Calculations

The ratio of the effective saturated hydraulic conductivity after PAM treatment ($K_{s\text{eff}}$) to that for the original intact soils ($K_{s1}$), $K_{s\text{eff}/K_{s1}}$, was measured using a one-dimensional saturated column in the laboratory (Moran, 2007; Young et al., 2009). The one-dimensional column experiments showed that, after PAM treatment, the soil column could be conceptualized as a system with two distinct layers. In this case, a thin layer of a mixture of PAM–sediment and other settling sediment, found on top of the soil, possessed a $K_s$ that was smaller than the original intact soil at the bottom of the column. The $K_s$ values reported in the laboratory experiments of Moran (2007) and Young et al. (2009) are thus the effective $K_s$ ($K_{\text{eff}}$, or $K_{s\text{eff}}$, or $K_{s2}$) of this two-layer system. Given that $K_{\text{eff}}$ for the one-dimensional, two-layer system is equal to the harmonic mean of the hydraulic conductivities of the respective layers (e.g., Freeze and Cherry, 1979), we can estimate the hydraulic conductivity of the PAM-coated layer, and express it as a ratio of hydraulic conductivities. Specifically, the measured $K_{s\text{eff}}$ is equal to the harmonic mean of $K_{s1}$ with a thickness of $(h_{\text{col}} - d)$, and $K_{s2}$ with a thickness of $d$, where $K_{s1}$ is the $K_s$ of the underlying soil, $K_{s2}$ is the $K_s$ of the PAM-treated layer, $h_{\text{col}}$ is the...
is the column height in the laboratory experiment, and \(d\) is the thickness of the PAM-treated layer. Using this relationship, we can calculate the ratio of the hydraulic conductivity of the PAM-treated layer to that of the original intact soils, \(K_{s2}/K_{s1}\):

\[
K_{s2} / K_{s1} = \frac{d}{(h_{col} - d)} \left( \frac{K_{s2}}{K_{s1}} \right)
\]

In the next step, we used Eq. [1] and the measured results of \(K_{s2}/K_{s1}\) from the saturated column experiments to develop a seepage ratio for a conceptualized simple, unsaturated two-layer soil system representing field canal conditions. We assumed that flow in the water delivery canal reaches equilibrium after some time and remains stable (i.e., no change in the canal water level or the level of the groundwater table), and that the thickness of the PAM-treated layer, \(d\), does not change with time; \(d\) has a very small magnitude and is a difficult variable to estimate. The parameter \(K_{s2}\) can also be combined with \(d\) to obtain the so-called hydraulic resistance, \(K_{s2}/d\) (e.g., Lado et al., 2007). Our development later indicated that the actual \(K_{s2}\) value (not the hydraulic resistance, \(K_{s2}/d\)) is needed to estimate the seepage reduction ratio. We further assumed that infiltration through the side walls of the water delivery canal is small compared with vertical infiltration, so the seepage is mainly one-dimensional. Using these assumptions, the flow process can be simplified as steady-state, one-dimensional vertical infiltration through a two-layer system of soils with distinct and contrasting hydraulic properties. Figure 1 shows a schematic of this conceptual model. This conceptualization also implies that horizontal water flow in the canal has a secondary effect on seepage losses, Darcy’s law gives (e.g., Zaslavsky, 1964; Warrick and Yeh, 1990) for water seepage. Also note that the spatial variability of hydraulic conductivity and other parameters was not considered in this study.

We now explain the general concept of a multiple-layer system of steady-state Darcy flow, and how it can be combined with the conceptual model described above to derive the seepage ratio. For steady-state conditions, Darcy’s law gives (e.g., Zaslavsky, 1964; Warrick and Yeh, 1990)

\[
z_{i+1} - z_i = \int_{\psi_i}^{\psi_{i+1}} \frac{d\psi}{1 + q/K(z)}
\]

where \(z\) is the distance above the groundwater table, increasing in the vertical upward direction, \(\psi\) is the suction head (a positive quantity for unsaturated soil), \(\psi = \psi_1\) at location \(z_1\) and \(\psi = \psi_{i+1}\) at location \(z_{i+1}\), \(q\) is the Darcian velocity (i.e., flux, positive upward), and \(K(\psi)\) is the hydraulic conductivity function.

For simplicity, the Gardner model (Gardner, 1958) is used to represent the hydraulic conductivity function

\[
K = K_s \exp(-\alpha \psi)
\]

where \(\alpha\) is the sorptive number. Other widely used hydraulic property functions should lead to a similar trend for the following analysis, although results may differ numerically.

Substituting Eq. [3] into Eq. [2] and integrating, we obtain

\[
z_{i+1} - z_i = \frac{1}{\alpha_{i+1}} \ln \left[ \frac{K_{s2} \exp(-\alpha_{i+1} \psi_{i+1}) + q}{K_{s1} \exp(-\alpha_{i+1} \psi_{i+1}) + q} \right]
\]

From Eq. [4], we can develop flow equations for the two layers separately. For soils below the PAM-treated layer, Eq. [4] can be simplified by taking \(i = 0\), and noting that \(z_0 = 0, \psi_0 = 0\) (i.e., the boundary condition at the groundwater table), and the boundary condition (\(z_1, \psi_1\)) at the interface between the PAM-treated layer and the canal bottom shown in Fig. 1:

\[
z_i = \frac{1}{\alpha_1} \ln \left[ \frac{K_{s1} + q}{K_{s1} \exp(-\alpha_1 \psi_1) + q} \right]
\]

Note that \(q\) is the vertical flux, which is the same for both layers (for infiltration, \(q\) is downward and therefore a negative quantity).

Similarly, by taking \(i = 1\) in Eq. [4], we can derive the equation for the PAM-treated layer (noting \(z_2 = L\), where \(L\) is equal to the groundwater table depth):

\[
L - z_i = \frac{1}{\alpha_2} \ln \left[ \frac{K_{s2} \exp(-\alpha_2 \psi_1) + q}{K_{s2} \exp(-\alpha_2 \psi_1) + q} \right]
\]

where \(L - z_1\) is the thickness of PAM-treated layer (i.e., \(d = L - z_1\)). Note that \(\psi = \psi_1\) at \(z = L\). In the case of ponding, \(\psi_2\) should be equal to the negative ponding depth, \(h\).

By algebraically rearranging Eq. [5], we can obtain

\[
\exp(-\alpha_1 \psi_1) = \left\{ \frac{[\exp(-\alpha_1 z_1) - \frac{q}{K_{s1}} \left[ 1 - \exp(-\alpha_1 z_1) \right] ]^{\psi_1/\alpha_1}}{\exp(-\alpha_2 \psi_1)} \right\}^{1/\psi_1/\alpha_1}
\]

Because in this study we dealt only with vertically downward infiltration, we used the convention that the infiltration rate is positive. Here \(p = -q\) is the infiltration rate, and the negative suction at the canal bottom is replaced with the canal water depth, \(h\), so that \(\psi_2 = -h\). Using these conventions and substituting Eq. [7] into Eq. [6], we obtain

\[
\exp(-\alpha_2 z_1) + \frac{p}{K_{s2}} \left[ 1 - \exp(-\alpha_2 z_1) \right] \left\{ \frac{\exp(-\alpha_2 \psi_1) + \frac{p}{K_{s2}} \left[ 1 - \exp(-\alpha_2 \psi_1) \right] }{\exp(-\alpha_2 \psi_1)} \right\}^{1/\psi_1/\alpha_1}
\]

Fig. 1. Schematic cross-section of a two-layer system of soils with distinct hydraulic properties underlying the canal; \(K_s\) is saturated hydraulic conductivity, \(\alpha_s\) is the sorptive number, \(z\) is the soil depth, \(L\) is the groundwater table depth, \(\psi\) is the suction head, and subscripts 1 and 2 refer to the original underlying soil and the surface layer treated with polyacrylamide (PAM), respectively.
where \( \beta = \alpha_2/\alpha_1 \), \( z_1 \) is the height of the native intact soils above the groundwater table, and \( z_1 = L - d \), where \( d \) is the thickness of the PAM-treated layer. When \( d \ll L \), then \( z_1 \sim L \) and the equation that governs the infiltration rate into the two-layer soil system shown in Fig. 1 now becomes

\[
\left\{ \exp(-\alpha L) + \frac{p}{K_{st}} \left[ 1 - \exp(-\alpha L) \right] \right\}^\beta + \frac{p}{K_{st}} \frac{1}{\delta} \left[ \exp(\beta d) - 1 \right] = \exp[\beta \alpha \left( d + h \right)]
\]  

\[\text{Eq. [9]}\]

where \( \delta = K_{st}^2/K_{st} \).

The infiltration rate, \( p \), can be solved iteratively from Eq. [9]. This equation should be relatively easily solved for \( p/K_{st} \), because the two terms on the left-hand side are both \( >0 \) and increase monotonically with \( p/K_{st} \). For the special case where \( \beta = 1 \), \( \delta = 1 \), and \( d = 0 \), Eq. [9] reduces to the simple one-layer system (i.e., flow through soil without a PAM-treated layer):

\[
\exp(-\alpha L) + \frac{p}{K_{st}} \delta \left[ 1 - \exp(-\alpha L) \right] = \exp(\alpha h)
\]

\[\text{Eq. [10]}\]

where the subscript WOP means without PAM treatment. Equation [10] can be solved for \( (p/K_{st})_{\text{WOP}} \), which results in

\[
\left( \frac{p}{K_{st}} \right)_{\text{WOP}} = \frac{\exp(\alpha h) - \exp(-\alpha L)}{1 - \exp(-\alpha L)} \frac{\exp[\alpha \left( h + L \right)] - 1}{\exp(\alpha L) - 1}
\]

\[\text{Eq. [11]}\]

For another special case where \( \beta = 1 \) and \( \delta < 1 \), Eq. [9] reduces to

\[
\exp(-\alpha L) + \frac{p}{K_{st}} \left[ 1 - \exp(-\alpha L) \right] + \frac{p}{K_{st}} \frac{1}{\delta} \left[ \exp(\alpha d) - 1 \right] = \exp[\alpha \left( d + h \right)]
\]

\[\text{Eq. [12]}\]

which yields

\[
\left( \frac{p}{K_{st}} \right)_{\beta = 1} = \frac{\delta \left\{ \exp(\alpha \left( d + h \right)) - \exp(-\alpha L) \right\}}{\delta \left[ 1 - \exp(-\alpha L) \right] + \left\{ \exp(\alpha d) - 1 \right\}}
\]

\[\text{Eq. [13]}\]

For more general cases, \( p/K_{st} \) from Eq. [9] needs to be solved iteratively. The ratio of the seepage rate for the two-layer soil system (i.e., PAM-treated soils) divided by that for the single-layer untreated soil system is defined as

\[
r = \frac{p/K_{st}}{(p/K_{st})_{\text{WOP}}}
\]

\[\text{Eq. [14]}\]

This ratio, referred to as the seepage ratio, represents a measure of the efficacy of PAM treatment for reducing water seepage loss. Hence, smaller \( r \) values correspond to more efficient PAM applications.

In summary, to compute \( r \), the values of the saturated hydraulic conductivity \( (K_{st}) \) and sorptive number \( (\alpha_1) \) of the original intact soils, the thickness of the PAM-treated layer, \( d \), and \( \beta \) (i.e., \( \alpha_2/\alpha_1 \)) are required. The procedure for calculating \( r \) from \( K_{st} \) consists of the following steps. First, \( K_{st}/K_{st} \) (i.e., \( \delta \)) is calculated from Eq. [1] based on the measured \( K_{st} \) in the column experiments. Second, the dimensionless seepage rate \( (p/K_{st})_{\text{WOP}} \) is calculated from Eq. [11] for the untreated soil. Third, the dimensionless seepage rate for the PAM-treated soils \( (p/K_{st}) \) is calculated from Eq. [9] iteratively. Finally, \( r \) can be calculated from Eq. [14].

### Sensitivity of Seepage Ratio to the Hydraulic Parameter Ratio

Given the difficulty in estimating the \( \alpha \) parameter for the PAM-treated top layer, the \( \beta \) values also cannot be determined. Nonetheless, we can investigate the sensitivity of the seepage reduction (ratio) to the \( \beta \) parameter by treating \( \beta \) as a random variable. Also, given the difficulty in measuring the thickness of the PAM-treated layer, \( d \), we used two estimated thickness values to examine the influence of \( d \) on the seepage ratio results.

Here, we examine the influence of two types of parameter distributions for representing the uncertainty of \( \beta \) (lognormal and uniform) on the sensitivity of \( r \). These were chosen because soil hydraulic parameters \( (K_s \) and \( \alpha_1 \)) are often found or assumed to obey lognormal distributions (e.g., Nielsen et al., 1973; Smith and Diekkruger; 1996; Leij et al., 2007). In this case, however, we were unable to validate whether \( \beta \) is also lognormally distributed; therefore, we included the uniform distribution for the sensitivity analysis.

For lognormal distribution, the probability density function is

\[
f_{ln} (\beta) = \left\{ \frac{1}{\sqrt{2\pi}\sigma} \exp\left[ - \frac{(\ln\beta - \mu)^2}{2\sigma^2} \right] \right\}
\]

\[\text{Eq. [15]}\]

for \( \beta > 0 \) otherwise

where the subscript \( \ln \) indicates the lognormal distribution. The parameters \( \mu \) and \( \sigma \) are related to the mean value of \( \beta \), \( \bar{\beta} \), and the coefficient of variation of \( \beta \), \( CV_\beta \), as follows (e.g., Zhang, 2002):

\[
\mu = \ln \sqrt{\frac{\bar{\beta}}{CV_\beta^2 + 1}}
\]

\[\text{Eq. [16]}\]

\[
\sigma = \sqrt{\ln \left( CV_\beta^2 + 1 \right)}
\]

\[\text{Eq. [17]}\]

For uniform distribution,

\[
f_{uf} (\beta) = \left\{ \begin{array}{ll} \frac{1}{b-a} & \text{for } a < \beta < b \\ 0 & \text{otherwise} \end{array} \right\}
\]

\[\text{Eq. [18]}\]

where the subscript \( uf \) indicates the uniform distribution. The parameters in the distribution \( a \) and \( b \) are related to \( \bar{\beta} \) and \( CV_\beta \) (e.g., Zhu et al., 2006):

\[
a = \bar{\beta} \left( 1 - \sqrt{5} CV_\beta \right)
\]

\[\text{Eq. [19]}\]

\[
b = \bar{\beta} \left( 1 + \sqrt{5} CV_\beta \right)
\]

\[\text{Eq. [20]}\]

Therefore, the mean seepage ratio and the mean squared seepage ratio for the lognormally distributed \( \beta \) are

\[
\bar{r}_m = \int_0^\infty r(\bar{\beta}) f_{ln} (\bar{\beta}) d\bar{\beta}
\]

\[\text{Eq. [21]}\]

\[
\bar{r}_m^2 = \int_0^\infty r^2(\bar{\beta}) f_{ln} (\bar{\beta}) d\bar{\beta}
\]

\[\text{Eq. [22]}\]

Similarly, the mean seepage ratio and the mean squared seepage ratio for the uniformly distributed \( \beta \) are
\[ r_{uf} = \int_a^b r (\beta) f_{uf} (\beta) d\beta \]  \[ \gamma_{uf}^2 = \int_a^b r^2 (\beta) f_{uf} (\beta) d\beta \]

Then, the coefficients of variation of the seepage ratio for the lognormal and uniform distributions can be calculated, respectively, as

\[ CV_{ln} = \sqrt{\frac{\gamma_{ln}^2 - (\mu_{ln})^2}{\mu_{ln}}} \]  \[ CV_{uf} = \sqrt{\frac{\gamma_{uf}^2 - (\mu_{uf})^2}{\mu_{uf}}} \]

The sensitivity analysis gives not only optimum predictions (i.e., mean predictions) of seepage efficacy, but also their associated uncertainty as measured by the CV.

As introduced above, three materials were used in laboratory experiments to investigate the mechanism of reducing \( K_s \) (Young et al., 2009; Moran, 2007). The particle size distributions for those materials are included in Table 1 and the graphical results are presented in Fig. 2, which illustrates the results of the \( K_{seff}/K_s \) term in Eq. [1] after PAM and suspended sediment treatment. As we can see from Fig. 2, the general trend is that the \( K_{seff} \) decreases with increases in both the PAM application rate (kg ha\(^{-1}\)) and the suspended sediment concentrations (mg L\(^{-1}\)). The biggest drop in \( K_{seff} \) is observed in the coarse-textured materials, and the smallest drop in \( K_{seff} \) is observed for the loamy sand soil. Under field conditions, variations in water temperature will also affect water viscosity and therefore seepage loss (Constantz et al., 1994; Doppler et al., 2007), but the viscosity change due to temperature variation is secondary to the effect of PAM accumulation, which results in an orders-of-magnitude decrease in the effective saturated hydraulic conductivity, especially in coarser grained materials, as shown by Young et al. (2009).

**RESULTS AND DISCUSSION**

Figure 3 plots the saturated hydraulic conductivity ratio (\( K_{s2}/K_s \)) of the PAM-treated layer to the untreated layer, which was calculated from Eq. [1], and Table 2 summarizes the parameter values used in the calculations. In the figure, we plotted the results for two estimated values for the PAM-treated layer thickness, \( d = 0.1 \) and 0.05 cm. While a distinct PAM-treated layer was clearly visible on top of the soil column in the earlier laboratory experiments, and the thickness of this layer was estimated on the order of <0.1 cm (Moran, 2007), an accurate measurement of the layer thickness was difficult. Nonetheless, we used two quite different estimates of PAM-treated layer thickness (0.1 and 0.05 cm) for calculating the \( K_s \) for the PAM-treated layer (\( K_{s2} \)). While the estimate of the thickness, \( d \), is rough, we will show below that the seepage ratio, \( r \), is not sensitive to \( d \). As expected, a smaller \( d \) value resulted in a smaller \( K_{s2} \) as indicated by the open symbol curves in Fig. 3.

Fig. 2. Measured effective hydraulic conductivity (\( K_{seff} \))/saturated hydraulic conductivity (\( K_s \)) ratio after polyacrylamide (PAM) and suspended sediment concentration (SSC) treatments for (a) ASTM C33 sand, (b) #70 mesh sand, and (c) loamy sand soil.

Fig. 3. Saturated hydraulic conductivity of the polyacrylamide (PAM)-treated layer (\( K_{s2} \))/saturated hydraulic conductivity of the underlying soil (\( K_s \)) ratio after PAM and suspended sediment concentration (SSC) treatment and different thicknesses of the PAM-treated layer (\( d \)) for (a) ASTM C33 sand, (b) #70 mesh sand, and (c) loamy sand soil.
Estimates of $\alpha_1$ are not available for the original soil materials used in the experiments, so typical values were taken from Carsel and Parrish (1988). Specifically, we used $\alpha_1 = 0.145$ cm$^{-1}$ for the ASTM C33 sand and $\alpha_1 = 0.036$ cm$^{-1}$ for the loamy sand soil. We used a value of 0.1 cm$^{-1}$ for the #70 mesh sand, which is between the sand and loam groups but is closer to the sand group. Figure 4 shows model results of the seepage rate ratio ($\beta$), calculated from Eq. [14], as a function of the PAM treatment level assuming a water table depth below the canal bottom of $L = 400$ cm and a canal water depth of $h = 50$ cm. The estimated thickness of the PAM-treated soil layer used in Fig. 4 is 0.1 cm. If other PAM-treated layer thicknesses are used, the seepage ratio results will be similar to the ones shown in Fig. 4, holding other parameters constant.

Table 3 shows the relative difference in the seepage ratio when $d = 0.05$ and 0.1 cm for conditions $L = 400$ cm, $h = 50$ cm, and $\alpha_1 = 0.145$ cm$^{-1}$. We can observe that the maximum difference is <2.5%, a relatively small difference given the two quite different thicknesses of the PAM-treated layer. The results indicate that the $K_{seff}$ obtained from the laboratory column experiments is a good indication of seepage reduction under conditions typically encountered in the field. Although there is some similarity between the laboratory column and the field condition in the sense that a thin PAM-treated layer exists in both situations, the major difference is that the bottom soil layer under the field condition is mostly unsaturated while the soil in the laboratory column is saturated. It is not surprising that the measured effective saturated conductivity in the saturated column is an indicator for seepage reduction from PAM treatment, but it is less obvious to what extent the thickness of the PAM-treated layer influences the seepage ratio for the two-layer system. The analysis from the simplified conceptualization of field conditions in this study indicates that the thickness of the PAM-treated layer does not significantly affect the seepage ratio.

It is important to note that we assumed that the thickness of the PAM-treated layer is constant with time, and that the associated $K_{seff}$ measurements also assume this constant thickness. The assumption was used given the difficulty in measuring the actual thickness of the layer. Nonetheless, for the purposes of this study, we show that the thickness of the PAM-treated layer will dictate the $K_t$ of that layer, as shown in Fig. 3.

Based on these results, we used an estimated PAM-treated layer thickness of 0.1 cm in our sensitivity analysis. The results and conclusions reported for this thickness should also hold for other estimated thicknesses of this layer.

The results in Fig. 4 show that, in general, PAM and SSC treatments are effective for coarse-textured soils, while treatment is not effective for reducing seepage loss for loamy sand. The finding is not surprising given that one mechanism that reduces $K$ from PAM treatment is the blocking of larger pores, with the potential for the formation of a thin PAM-rich layer, as observed by Lentz (2003). The hydraulic conductivity of the PAM-treated layer and that of the untreated loamy sand soil are not very contrasting because relatively few large pores exist in this soil, even when untreated. As a result, PAM treatment is not effective for reducing seepage loss for loamy sand. The results in Fig. 4 illustrate that the efficiency of both PAM and SSC is not linear; rather the seepage ratio decreases significantly for PAM treatment levels between 0 and 22.4 kg ha$^{-1}$ for coarse-textured soils, but the reduction efficiency diminishes when PAM treatment is increased to 44.9 kg ha$^{-1}$, as seen by a stabilization of the seepage ratio.

Figure 5 shows the mean and CV of the seepage ratio, calculated using Eq. [21], [23], [25], and [26], as they relate to PAM treatment levels for the ASTM C33 sand when $L = 400$ cm, $h = 50$ cm, and $d = 0.1$ cm. The mean value of $\beta$ is 1.0, with CV values for $\beta$ of 0.2 (Fig. 5a) and 0.4 (Fig. 5b). Note that for both lognormal and uniform distributions, we used the same mean and CV values. As the CV of $\beta$ increases, the mean seepage ratio for both lognormal and uniform distributions also increases, as can be seen from a comparison between Fig. 5a and 5b. Due to the nonlinear relationship between $\beta$ and the seepage ratio, the seepage ratio calculated using the mean $\beta$ value is different and typically lower than the mean seepage ratio. Uncertainty in $\beta$ is propagated and augmented significantly into the uncertainty of the seepage ratio. As seen in the bottom plots in Fig. 5, the CV of the seepage ratio could be as high as almost 3 for the C33 sand where the CV of the input $\beta$ is only 0.2, which means the seepage ratio is sensitive to and significantly influenced by $\beta$. The CV of the seepage ratio is higher when $\beta$ is represented by a lognormal distribution than when represented by a uniform distribution. Since
the lognormal distribution is positively skewed, the probability of sampling large values of β is higher than for the uniform distribution. A few high β values could significantly increase the uncertainty of seepage ratio prediction. While the predicted CV of the seepage ratio for the lognormal distribution case is similar for CV_β = 0.2 and 0.4, the actual standard deviation of the seepage ratio is still higher for CV_β = 0.4 given that the mean seepage ratio is higher for the case of CV_β = 0.4 than for CV_β = 0.2.

Figure 6 shows the mean and CV of the seepage ratio as functions of PAM treatment levels for the #70 mesh sand when L = 400 cm, h = 50 cm, and d = 0.1 cm. The results are quite similar to those presented for the C33 sand, with one noticeable difference. The CV of the seepage ratio based on a lognormal distribution of β is much lower when CV_β = 0.2 than when CV_β = 0.4. No similar differences were observed for the C33 sand, which indicates that the uncertainty in β is propagated and augmented more significantly into the uncertainty in the seepage ratio for the coarse-textured C33 sand, while for the relatively finer textured #70 mesh sand (reflected in the α values used) the uncertainty propagation and augmentation are relatively gradual. This finding calls for a more urgent need to quantify (or estimate) the α parameter for the PAM-treated soil layer for the coarse-textured sand, because any uncertainty in β (i.e., the α ratio) will translate into a larger uncertainty when predicting the seepage ratio for coarse-textured soils, where PAM treatment is more likely to occur.

Figure 7 shows the mean and CV of the seepage ratio for the loamy sand for the same experimental conditions as used in Fig. 5 and 6. For the loamy sand, the effect of PAM treatment on reducing K_s is small (see Fig. 2), which means that K_s values for the two layers are not contrasting. As a result, the value and uncertainty of the seepage ratio are less sensitive to PAM treatment and SSC value.

Table 3. Relative difference in seepage ratio results between thicknesses of the polyacrylamide (PAM)-treated layer of 0.05 and 0.1 cm, for a groundwater table depth of 400 cm, a canal water depth of 50 cm, a sorptive number for the untreated soil of 0.145 cm⁻¹, suspended sediment concentration (SSC) treatments of 0, 150, or 300 mg L⁻¹, and hydraulic parameter ratios (β) of 0.5 or 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>PAM</th>
<th>SSC = 0, β = 1</th>
<th>SSC = 150, β = 1</th>
<th>SSC = 300, β = 1</th>
<th>SSC = 0, β = 0.5</th>
<th>SSC = 150, β = 0.5</th>
<th>SSC = 300, β = 0.5</th>
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<tr>
<td></td>
<td>kg ha⁻¹</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>−0.065</td>
<td>0.151</td>
<td>0.377</td>
<td>−0.088</td>
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<td>0.418</td>
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<td>−0.162</td>
<td>−0.275</td>
<td>−0.062</td>
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<td>−0.181</td>
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<td>−0.080</td>
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<td>−0.034</td>
<td>−0.064</td>
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<tr>
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<td>−0.120</td>
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CONCLUSIONS

Polyacrylamide treatment is more effective for reducing seepage when coarse-textured materials underlie canals. The results show that β, the sorptive number ratio in these two-layer systems, is a sensitive parameter for estimating the seepage loss from coarse-textured, unlined water delivery canals, and that uncertainty in the sorptive number ratio significantly augments uncertainty in the seepage ratio.
While accurate determination of the thickness of the PAM-treated layer is not essential for examining the effectiveness of PAM treatment, an estimate of the sorptive number $\alpha$ for the PAM-treated layer was found to be more important. Therefore, reasonable estimates (laboratory or field determination) of the $\alpha$ parameter values of the PAM-treated layer are recommended; however, new methods probably will be needed to better quantify this parameter, as we are unaware of methods available to estimate $\alpha$ in this type of hydrologic system.

The focus of this study was on the seepage ratio of treated soils to untreated native soils, not the actual seepage rate. When comparing the results for different soil types, caution should be used in interpreting the results. A smaller seepage ratio for coarse-textured soils compared with finer textured soils simply means that the PAM application is more effective for the former in terms of reducing seepage losses. The results do not imply that overall seepage loss is smaller for the coarse-textured soils than for the fine-textured soils.

Although the numerical results presented here are based on the previous study by Young et al. (2009), the framework proposed in this study is independent of any particular data set. The procedure can be applied to incorporate other laboratory experiments of similar type. Finally, while this study focused on the conceptualization of a two-layer system as a result of PAM treatment on an unlined canal, the basic idea is quite generic to similar systems and could be applied independent of PAM treatment.

ACKNOWLEDGMENTS

The funding support by the U.S. Bureau of Reclamation, under Cooperative Agreement no. 05-FC-81-1165, is greatly appreciated. Del Smith is the program manager. We also thank Ernesto Moran for providing laboratory testing results of saturated hydraulic conductivities and many helpful discussions.
REFERENCES


