Fractures in geologic media are known to provide preferential pathways for fluid flow and transport of contaminants. Computations of flow and transport in fracture networks are usually performed using either a discrete fracture network (DFN) modeling approach or a continuum based approach. The DFN approach explicitly models individual elements of a network and thus is considered to provide accurate estimates of flow and transport. The limitations of DFN method arise from computational constraints in both processing speed and memory, and difficulties in incorporating fracture-matrix interactions. Continuum based approaches assign equivalent hydraulic parameters, derived from statistical properties of the network, to cells of a continuum mesh. Such methods allow for simulation of processes not presently achievable using DFN methods, but can suffer from lack of accuracy especially in predictions of transport behavior of the network. Applying a continuum based approach becomes more challenging for sparse fracture networks where preserving the connectivity of elements and anisotropic behavior of the network is of paramount importance. We present a set of mapping rules and upscaling techniques to develop an improved fracture continuum methodology. The method requires an anisotropic conductivity field for the fracture continuum grid as opposed to assigning a scalar value for each cell. Comparisons with DFN simulations demonstrate the accuracy of the improved fracture continuum method in simulating both fluid flow and transport characteristics, including early- and late-time tails, for a wide range of fracture density values and grid cell sizes.

INTRODUCTION

Fluid flow and transport in fractured rock is controlled by networks of interconnected conductive fractures. Spatial distributions of geometrical properties of fracture network and their hydraulic properties give rise to clusters of fractures linked together to form the hydraulic backbone connecting opposite sides of a domain of study. Usually very little information is available regarding fractures in the subsurface as only those fractures that are visible in outcrops or intersect well-bores in fields can be observed. Indirect information about fractures can also be obtained from seismic data, and well testing etc. This information is used to generate probabilistic models of fracture parameters, such as distribution of length, location, orientation, permeability, and fracture density. The stochastically parameterized fracture networks are then solved for flow and transport, which may include exchange of mass between fractures and matrix depending upon properties of the matrix continua such as porosity and permeability. Given the enormous number of fractures that can be present in a field, and the degree of variability in both geometrical and hydraulic properties of the network, the modeling process is typically very demanding from a computational point of view.

Two common numerical techniques to study fracture networks are the discrete fracture network (DFN) modeling approach and the stochastic continuum (SC) approach. The DFN models provide a means for explicitly representing the flow path geometry, such that the pattern of interconnection among fractures determines the movement of water and solute [Priest, 1993; Sahimi, 1995]. Flow in a DFN model is calculated in each fracture (represented as lines in two-dimensions and planar discontinuities in three-dimensions) individually, and hence, there is no need to represent a fracture with an equivalent grid cell conductivity. DFN methodologies represent potentially the most accurate way of studying flow through fractured rocks. However, owing to the computationally intensive nature of calculating flow and transport through intersection points of thousands of fractures, the applicability of DFN model is limited to small scale problems. Additionally, further modeling complexity arises when processes related to fracture-matrix interactions are to be simulated. The SC approach is based on the conceptualization of treating fractured rock as equivalent porous medium described in terms of physical parameters (conductivity, specific storage, etc) that vary in space according to spatially varying random functions. For two-dimensional problems, the fracture network is divided into grid cells and each cell is assigned an equivalent
conductivity based upon the network of fractures within that cell [Jackson et al., 2000; Niemi et al., 2000]. Though less computationally intense than DFN models, the SC models suffers from its own drawbacks such as the problem of estimating representative elementary volume (REV) on which the fractured rock mass can be said to behave as an equivalent porous medium. Indeed, some of the commonly used techniques to compute upscaled permeability tensor for grid cell rely on the assumption of high value of fracture density [Oda, 1985]. The SC models thus runs into pitfalls when density of fracture networks does not lead to a well defined REV. Assigning an equivalent permeability tensor to each cell of a grid also tends to overly homogenize the representation of fracture networks, and these types of simulations may therefore be unable to reproduce extreme flow and transport behavior.

Fracture continuum (FC) approaches that combine the merits of DFN and SC models have been developed [Svensson, 2001; Botros et al., 2008, Reeves et al., 2008]. In essence, the FC methods use the mapped geometry of fracture networks on a finite-difference grid to compute the cell conductivities in such a way so that the total flow across the domain is preserved. The existing FC methods primarily are designed for flow computation only and have been found to usually overestimate the network flow as compared to the original DFN solution due to increased artificial connectivity in the grid. We present herein a significantly improved FC methodology based on the techniques originally presented in Botros et al. (2008) and Reeves et al. (2008). These mapping and upscaling methods not only results in more accurate computation of flow (when compared to DFN solution) but also ably predicts the transport characteristics for a wide range of network properties and grid cell sizes. Groundwater flow models such as MODFLOW can utilize these techniques to perform more efficient simulations in fractured media.

**METHODOLOGY**

The approach described in this paper can be viewed as an upscaling procedure in which we map the fine scale (discrete fracture) model onto a coarse scale (continuum) model. We consider the fracture network to be well connected over the domain of study and the fracture permeability is assumed to be large compared to matrix permeability. This allows us to compute flow and transport as a single-continua process and directly compare solutions from the DFN and FC methods. Transport is modeled as a purely advective process (without a local dispersion component) in individual fracture segments. Observed dispersion in breakthrough curves is attributed to the tortuosity of fracture network, and variable rate of particle movement owing to the variance in the conductivity distribution.

The fracture networks are stochastically generated using user defined (presumably derived from field data) statistical distributions of geometric and hydraulic properties. The network is simplified by identifying and isolating non-conductive fractures, and the DFN solution for flow and transport is then obtained. Mapping the network on a finite-difference grid and application of rules of our FC methodology yields anisotropic conductivity values for each grid cell. Assuming steady state and saturated conditions, flow and transport is solved on the finite-difference grid and is then compared to the DFN solutions to examine the efficiency of our mapping and upscaling methodology.

**Solving Flow and Transport using a DFN model**

Stochastic fracture network is generated by incrementally adding fractures with random location, length and orientation until a specified density (defined as sum of all fracture lengths divided by the domain area) criterion is reached. For this study, the fracture length $L \in [L_{\text{min}}, \infty)$ is modeled as a power-law with an exponent $a \in [1,3]$ where the probability density function is given as

$$f(L) = \frac{aL^a}{L_{\text{min}}^{a+1}}.$$  \hspace{1cm} (1)

Low values of the power-law exponent lead to the generation of longer fractures and a well connected network for a given value of density. The locations of fracture centers are assumed to be uniformly distributed over the computational domain. The fracture orientations are modeled according to von Mises-Fisher distribution functions which may be thought of as circular analogue of the normal distribution. We generate two sets of fractures differing in their mean orientation by a large angle so that the resulting network has a high degree of connectivity between the fractures. Conductivity values are assigned to
fractures as independent random variables selected from a log-normal distribution. Flow through the generated fracture network (Fig. 1a) is then solved via DFN techniques. This entails prescribing a boundary condition (we use fixed hydraulic-head values on the top and bottom boundaries and linearly decreasing hydraulic-head values on the side boundaries), and solving for head values in the interior of domain at all intersection points of fractures. As has already been mentioned, DFN models are computationally intensive, and hence, it’s beneficial to reduce the number of unknowns (intersection points of fractures) by identifying and deleting non-conductive fractures. This is achieved by sequentially numbering end-points (nodes) of all fracture segments and then scanning the domain to filter out segments that has an end-point unconnected to any other node in the domain. Note that a fracture segment is defined here as the span of an individual fracture between two successive intersection points. Therefore a long fracture intersected by many other fractures is modeled as a collection of many smaller segments. The above process of identifying non-conductive fractures, which can either be an isolated fracture or a dead-end segment, is iterative. This is because the deletion of a dead-end segment can turn other segments in the network into an isolated fracture or a new dead-end segment. Careful application of this identification and deletion process results in a simpler network with significantly less number of intersection points (Fig. 1b). A further simplified network (Fig. 1c) can be obtained as a by-product of the flow solution by identifying segments with negligible flow as an ad-hoc way of deleting isolated fractures.

Solving for flow in a DFN model is an exercise of finding the head value at all nodes inside the domain. Assuming Darcy’s law govern flow in a fracture, we write continuity equation (incoming flow = outgoing flow) for all nodes which gives rise to a set of linear algebraic equation in head. The resulting coefficient matrix is highly sparse with an irregular sparsity pattern. An iterative solution method for sparse linear systems, appropriately chosen for these classes of problems, is then applied to find the solution. Breakthrough curves for conservative, non-diffusing particles are obtained by releasing particles from a source zone close to the top boundary and tracking them from one internal node to the next in a Lagrangian framework until they reach one of the exit boundaries. At nodes where more than one possible direction is available, particles are randomly assigned direction based on the relative flux values.

Mapping rules for computation of flow on the continuum grid

The flow along a fracture, represented as a linear discontinuity on a two-dimensional plane, is estimated using the Darcy’s law with the assumption that the width of network in the third dimension is unity. If $L$ is the length of the fracture, $T$ its transmissivity, and $dH$ is the difference in heads between the fracture ends, then the flow is estimated as

$$Q = T \frac{dH}{L}. \quad (2)$$

Assuming the fracture network is mapped onto a grid of cell size of $\Delta$, the value of hydraulic conductivity for a cell intersected by a fracture of transmissivity $T$ can be estimated by the ratio $T / \Delta$. This estimation needs further correction in cases of inclined fractures as has been explained in Botros et al. (2008) and Reeves et al. (2008). Mapping an inclined fracture onto a grid results in longer flow path since water
molecules travels to adjacent cells strictly in a horizontal or vertical fashion. This gives rise to the so-called “stair-step” pattern where the total length traversed is more than the fracture length $L$. Thus a decrease in hydraulic gradient occurs between two end-points of an inclined fracture which needs to be accounted for by increasing the cell hydraulic conductivity so that the amount of flow is preserved. The modified expression for cell hydraulic conductivity for a fracture inclined at an angle $\theta$ with the $x$-direction is calculated as

$$K = \frac{T}{\Delta} C(\theta),$$

(3)

where $C(\theta) = |\sin(\theta)| + |\cos(\theta)|$ is the correction factor introduced to compensate for the decrease in head gradient as a result of mapping. Botros et al. (2008) uses this correction factor selectively for cells that fulfill a density criterion. Moreover, the conductivity value is assigned in only one direction if a fracture intersects both faces of a cell in that direction, and is assigned in both directions otherwise. It’s observed that the mapping methodology and correction factor proposed by Botros et al. (2008) helps in achieving fairly good estimates of global flow values (when compared to DFN solution) for small cell size grids, but the discrepancy increases rapidly with increasing cell size and/or increasing variance of the conductivity distribution. It appears that the mapping rules tends to overestimate amount of flow between neighboring cells most likely because of enhanced conductivity values.

We therefore present here some simple additional rules that serve to limit the chances of flow occurring between cells not directly linked by a fracture, and reduce the correction factor assigned to short fractures as they may not have a span long enough to induce a “stair-step” pattern. Unlike Botros et al. (2008), we apply correction factor to all fractures regardless of how dense a cell might be. It should also be noted that mapping is performed not using the entire network of fractures (Fig. 1a) but by using only those fractures which form a conductive backbone (Fig. 1b or 1c). The rules of mapping a fracture network on a continuum grid are:

1. Conductivity values in all four directions ($x^+, x^-, y^+, y^-$) of a cell are computed separately. If a fracture doesn’t intersect a face, its contribution to the cell conductivity in direction of that face is nil. Thus a cell intersected by fracture(s) only at the top and right face will have conductivity value of zero in the $-ve$ $x$ and $y$ directions.
2. Correction factor of $C(\theta)$ is assigned as per Eq. (3) when fractures longer than twice the cell size intersect one of the four faces of a cell.
3. For short fractures (smaller than twice the cell size), an argument can be made that the “stair-step” pattern of flow on a grid will quite likely be absent. Instead, flow may occur strictly vertically or horizontally on the grid. Hence the path on the grid could possibly become shorter than the actual fracture length. Thus the correction factor applied is just equal to $|\sin(\theta)|$ when the fracture intersects either top ($y^+$) or the bottom ($y^-$) face of the cell, and becomes equal to $|\cos(\theta)|$ otherwise. In a way this modification corroborates the observation of Botros et al. (2008) that the correction factor for dense cells (likely because of presence of many smaller fractures) is small and perhaps close to unity.
4. Transmissivity (modified appropriately for the inclination) of all fractures intersecting a particular face of a cell are added together and then divided by the cell size (Eq. (3)) to give the conductivity value of the cell in direction of that face.
5. Flow between adjacent cells is governed by the harmonic mean of their respective conductivity values. This rule not only honors a basic outcome of flow in series, but also ensures that probability of artificial connectivity on the grid is minimized (since harmonic mean is closer to the smaller of two numbers).

**Modifications for computation of transport on the continuum grid**

The rules presented above also pave the way for better prediction of transport characteristics as the important factor of minimizing the likelihood of artificial connectivity is achieved for small to moderate cell sizes. However, the “stair-step” pattern in conjunction with the correction factor introduced in Eq. (3) tends to delay the arrival of particles when compared to transport solution from a DFN model. To address this anomaly, the time it takes for particles to travel between centers of a pair of adjacent cells is multiplied by a transport correction factor (less than unity):
\[ C_r(K, \theta) = \sum_{i=1}^{N} K_i \left/ \sum_{i=1}^{N} K_i C_i(\theta) \right. \]

where \( N \) is the total number of fractures intersecting a given face of a cell. For a single fracture, Eq. (4) reduces to \( 1/C(\theta) \). Just like the conductivity values, \( C_r(K, \theta) \) can potentially be a different value for each of the four faces of a cell. When particles migrate to a neighboring cell, the average of the correction factor between the cells is used.

**SUMMARY**

The rules and techniques laid out in the previous section were applied to a variety of fracture networks encompassing a wide range of length and conductivity distribution. The cell sizes were varied with respect to the minimum fracture length and various values of density were tested. It was observed that the global flow values as computed from the DFN and the improved FC models showed remarkable agreement for various sets of geometrical and hydraulic parameters (see Fig. 2). The values differed significantly only when the cell size became a couple of times larger than the minimum fracture length. Transport simulations, which are run in Monte-Carlo framework, demonstrated marked agreement between the models for majority of realizations. Breakthrough curves as computed using the FC model usually is successful in reproducing tailing behavior of the DFN solution (Fig. 2). In summary, the improved FC methodology as presented in this paper provides a robust tool to study flow and transport in fractured media. The method honors the anisotropic behavior of a fracture network and successfully implements rules to minimize artificial connectivity on a continuum grid.

![Global Flow comparison for 10m Cell Size](image1)

![Breakthrough comparison for 10m Cell Size](image2)

*Figure 2. Representative plots of comparison of global flow values (left) and breakthrough curve (right) between a DFN and a FC model. Cell size is 10m for a 1kmx1km domain of moderately dense fracture network.*

**REFERENCES**


