Sediment residence time distributions: Theory and application from bed elevation measurements

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[1] Travel distance and residence time probability distributions are the key components of stochastic models for coarse sediment transport. Residence time for individual grains is difficult to measure, and residence time distributions appropriate to field and laboratory settings are typically inferred theoretically or from overall transport characteristics. However, bed elevation time series collected using sonar transducers and lidar can be translated into empirical residence time distributions at each elevation in the bed and for the entire bed thickness. Sediment residence time at a given depth can be conceptualized as a stochastic return time process on a finite interval. Overall sediment residence time is an average of residence times at all depths weighted by the likelihood of deposition at each depth. Theory and experiment show that when tracers are seeded on the bed surface, power law residence time will be observed until a timescale set by the bed thickness and bed fluctuation statistics. After this time, the long-time (global) residence time distribution will take exponential form. Crossover time is the time of transition from power law to exponential behavior. The crossover time in flume studies can be on the order of seconds to minutes, while that in rivers can be days to years.


1. Introduction

[2] As in many fields with stochastic approaches to particle transport, the conceptual model used to represent sediment transport through time in gravel bed rivers equates the distribution of overall travel distance at time \( t \) of a single particle as the sum of a random number of randomly sized step lengths. This random walk concept is chosen because, although the physics of particle motion is not random, we are currently only able to measure the bulk properties of moving particles rather than predict exact travel distances or times. These bulk properties take the form of probability densities, which give the likelihood of the set of possible step length distances in a given time step. We assume that the transport statistics (mean travel distance and the spread about the mean) of a single particle represent the actual distribution of a cloud of particles released in a channel at the same time. While it is possible to include a zero-travel distance due to particle deposition on or within the bed in the collection of likely travel distances, it seems that sediment residence (also waiting or rest) time in the bed plays such a significant role in overall transport that from a modeling perspective, particle motion characteristics and sediment residence time in the bed must be considered individually. Residence time accounts for the difference between instantaneous particle velocity and streamwise virtual velocity, which has been observed to decrease through time [Ferguson et al., 2002; Ferguson and Hoey, 2002; Haschenburger, 2011]. Consideration of sediment residence time in the bed of stream channels is useful on its own because it impacts aqueous chemical exchange and controls the storage and release of contaminants and nutrients in the river. Sediment residence time in channel substrate is a function of entrainment physics, bed morphology, and characteristics of vertical mixing in the bed, but our inability to directly measure the forces acting on individual grains leads to characterization of particle immobile periods by residence time probability distributions [Hassan et al., 2013].

[3] There is a large class of probability models that are used to predict transport properties given a jump length distribution and a residence time distribution. One general model, known as a continuous time random walk (CTRW), incorporates many of the stochastic models of sediment transport proposed since Einstein [1937] (details below) and can be used one of two ways. First, if the exact jump length and residence time distributions are known, then the analytical form of their densities can be plugged into a master equation that produces the probability density describing travel distance through space and time [Scher and Lax, 1973]. Second, if only the tail properties of the jump length and residence time distributions are known—that is, whether the tails of the distributions decay at exponential or power law rates—then asymptotic (long-time) travel distance can
be computed. In either case, particle transport that is well
described by discrete CTRW models is governed in the long-
time limit by advection-dispersion equations (ADE). The
difference between physical regimes that are modeled using
preasymptotic or transient equations versus long-time asymptotic
equations has been identified for sediment transport using the
terms “local,” “intermediate,” and “global” diffusion re-
gimes [Nikora et al., 2001; Nikora et al., 2002]. That work
calls for experimental and theoretical evaluation of transition
between the regimes [Nikora et al., 2002].

[4] Radio transmitter tagged stones [Ergenzinger et al.,
1989; Habersack, 2001], magnetic stones, and passive
integrated transponder tagged stones [Lamarre et al., 2005;
Bradley and Tucker, 2012] have been used in Lagrangian
analysis of resting periods, where consecutive rests of a
single particle are tracked. Recent technology used to measure
bed topography such as sonar transducers and light detection
and ranging (lidar) allows collection of bed elevation time
series at a point or in the horizontal plane. While data
describing particle step length and velocity characteristics
are readily produced in both flume and field gravel tracer
tests, sediment residence times are more difficult to record.
As a result, residence time is frequently backed out from
overall tracer transport characteristics rather than directly
measured. The first goal of this study is to use bed elevation
data to measure sediment residence time distributions
(and entrainment probabilities) explicitly through labora-
tory measurements and describe appropriate theoretical
residence time distributions for use in analytical models of
particle transport.

[5] Some notes on the significance of certain types of proba-
bility distributions in stochastic models of particle transport
will put historical studies in context. It has long been known
that a stochastic description of gravel dispersion must be a
function of a relative measure of time, where total time is scaled
by the mean discrete time at rest [Stelczer, 1981, p. 177].
However, in recent decades, the significance of exponential-
type (thin tailed or finite mean) versus power law (heavy tailed
or infinite mean) residence time distributions in determining
existence of an average time at rest has become clear
[Schumer et al., 2009]. The effect of exponentially distrib-
uted residence time, or of any residence time distribution
with well-defined (i.e., finite) mean (such as a gamma,
Gaussian, or any truncated distribution), is to retard virtual
velocity of sediment relative to the instantaneous in-motion
velocities. If a collection of tracers is released into a channel,
their velocity distribution will reflect the fact that the
particles begin in motion. As particles deposit and reside
on and within the bed, incorporation of zero velocities will
lead to an exponential decrease in overall travel velocity.
Eventually, as the longest possible residence times are
encountered and an average residence time emerges, the
overall average transport velocity, or virtual velocity \(V_r\),
will be related to the average mobile velocity \(v\) scaled by a
retardation coefficient \(V_r = v/r\), where \(r\) is set by the average
residence time in the bed. If a dispersion coefficient is used
to describe the distribution of velocities around the average,
it will also be scaled by the retardation coefficient. Certain
heavy-tailed power law distributions on the other hand do
not produce samples that converge to a mean value because
the high probability of extreme values leads to subsequent
values that continually throw off the sample mean. If residence
times have an infinite-mean power law distribution, there is
no convergence to a long-term average residence time and
the virtual velocity will continue to decay as a power law.
The latter model is unsatisfying as it implies that all parti-
cles will eventually be immobile—an unattractive idea for
a nonaggrading bed. Instead, it is more realistic to consider
that if residence times vary over many orders of magnitude
as a power law, it would only be up to some finite value,
likely related to the depth of the bed. This would result in
power law decay of the virtual velocity until the incorporation
of a maximum residence time finally leads to emergence of an
average residence time. These types of residence time distribu-
tions can be represented with truncated or tempered power
law distributions [Zhang, 2010; Zhang et al., 2012] which
have power law character up to a cutoff after which the
distribution decays much more rapidly. The limiting, con-
tinuum analytical equations that are associated with random
walk-type models are different depending on the type of
residence time distribution. Exponential-type residence
times combined with diffusive transport lead to classical
advection-dispersion equations, while infinite-mean power
law residence times lead to fractional-in-time advection-
dispersion equations [Schumer et al., 2003]. Finally, tempered
fractional ADEs can reproduce the features of transport when
power law residence times up to a finite maximum exist
[Zhang et al., 2012]. The transient (or preasymptotic) period
before an average residence time is achieved may produce
decay in virtual velocity with exponential or power law type
decay. If the long-term average sediment residence time is
not reached within the time of observation or model period
of interest, then the virtual velocity appears transient and will
continue to decay. The choice of transport model depends on
whether the timescale of interest is within the transient period,
the asymptotic period, or both.

[6] Early studies on sediment residence time included
development of stochastic models that fit laboratory flume
experiments incorporating painted tracer stones [Einstein,
1937]. It was observed that travel time of a stone was small
relative to its resting period between movements so that
overall travel could be modeled using alternating random
jump lengths and random resting periods. The residence

time distributions used to describe the collection of resting
periods were assumed to be exponential based on conceptual-
thoretical grounds but not on resting period data. Stelczer
[1981] argued that in practice, Einstein did not actually treat
resting time as a random variable, but as a constant. In decades
to follow, the Einstein formulation was the basis for models of
flume and field tracer transport and the assumption that
sediment residence times are exponential persisted [Hubbell
and Sayre, 1964; Yang and Sayre, 1971; Sun and Donahue,
2000]. Use of a compound Poisson Process as a stochastic
model for sediment motion [Hubbell and Sayre, 1964], for
example, defined particle travel distance as the sum of jumps
of random length and by definition included exponentially
distributed resting periods between jumps so that the number
of steps that occur during a discrete time interval can be
described by a Poisson distribution [Ross, 1997]. In the scaling
limit, particle motion described by a classical ADE. Interestingly,
Hubbell and Sayre [1964] stated that they are not sure if the exponential
resting periods assumption is valid and leave open the possi-
bility that this assumption will be invalided by experiments.
An example of exponentially distributed residence time data in the literature exists for a plastic particle in a sand-bed flume [Yang and Sayre, 1971]. Our own review of a set of residence time data from Habersack [2001] suggests that the author used a qualitative fit to an exponential function rather than a distributional goodness of fit compared with other classes of distributions. We find power law resting times over the Habersack [2001] experiment up to 30 min, with a crossover to exponential resting times between 50 and 90 min (Figure 1). Recent flume experiments have documented power law residence time distributions and associated them with long-term subdiffusive transport [Martin et al., 2012]. There is also evidence that power law residence times occur in natural channels. For example, Nikora et al. [2002] proposed a conceptual model describing variance behavior ($\sigma^2 \propto \tau^\gamma$, where $\gamma$ is a scaling exponent) for particle plume growth across three distinct scales, where variance represents dispersive spread around average growth behavior. Significantly, at long timescales, particle travel distance was expected to be subdiffusive ($\gamma < 0.5$) because of a wide distribution of sediment residence times. Gravel tracer data from field experiments supported this hypothesis at a variety of timescales [Nikora et al., 2002; Drake et al., 1988]. However, others [Zhang et al., 2012] argued that subdiffusive sediment transport in channels will only exist until the long-time average residence time is reached. They supported this argument by showing improved model fits to the classic Sayre and Hubbell [1965] data when a transport model incorporates a tempered Pareto residence time distribution in which power law decay of residence times becomes exponential after some characteristic averaging period is exceeded. While the first goal of this study (stated above) is to measure sediment residence time distributions, the second goal is to describe the origins of transient power law residence times in a bed and our expectations for the transition time to exponential residence times.

Figure 1. Residence time distribution from a radio-tracked gravel field experiment [Habersack, 2001, digitized from Figure 5] showing distinct decay behavior: the left section of the curve has a power law decay rate, and the right section of the curve has an exponential decay rate. A hump, characteristic of transition periods, appears between the power law and exponential portions of the distribution.

The remainder of this manuscript is organized as follows: we first describe translation of bed elevation time series into resting periods of the many particles deposited and entrained at a single location in the bed through time and develop an empirical model for overall sediment residence time distributions (section 2). Next, we discuss the theory of how residence time is linked to bed elevation for two cases: a hypothetical case using semi-infinite bed elevation and a realistic case using finite bed elevation (section 3). We derive sediment residence time distributions from two examples of bed elevation series from flume experiments: a simple case with homogeneous sediment under low flow conditions on plane-bed morphology [Wong et al., 2007] and a more complicated case with heterogeneous sediment under high flow conditions on large-scale bedform morphology [Singh et al., 2009] (section 4). Both cases are short duration experiments of several minutes to several hours of runtime. Finally, we discuss implications for modeling sediment residence time from a bed elevation series.

2. Empirical Model of Residence Time

It is difficult to continuously track the location of buried particles to determine individual sediment residence times, but with the advent of continuous bed elevation tracking, it is now possible to measure exactly unconditional residence time and entrainment probability at a given location in a stream or flume for the many particles that come to rest on or within the bed. Key is to equate time of deposition with the time that a fluctuating bed moves upwards and time of entrainment with the time that a bed moves downwards. Consider a bed whose solid bottom is at reference elevation $z = 0$. The elevation of a stable bed will fluctuate between zero and a maximum elevation $L$ set by sediment load, flow regime, and hydraulic drag at the channel surface [Knighton, 1998]. Suppose that we discretize bed thickness into a set of $n$ bed elevations $z_i$ for $i = 1, \ldots, n$. The residence time between particle deposition...
and entrainment at an arbitrary elevation (including the surface) $z_k$ is the time between two events [Yang and Sayre, 1971; Nakagawa and Tsujimoto, 1980]:

[10] 1. Deposition of a particle at $z_k$, which occurs when the bed elevation moves above $z_k$

[10] 2. The entrainment of the particle at $z_k$, which occurs when the bed elevation moves below $z_k$ from above

[11] Note here that any particle that comes to rest on the bed is conceptually part of the bed, whether it is buried or on top of other grains. The conditional distribution of residence times at elevation $z_k$ is the collection of residence times $R_i$ collected from the bed elevation series as defined above (Figure 2a). Development of empirical residence times from bed elevation fluctuations has limitations related to measurement interval in both time and space. First, if equilibrium transport occurs, such that the rate of particle entrainment is balanced by the rate of deposition and the bed elevation does not change over timescales greater than the measurement interval, then any entrainment and disentrainment events will not be recorded. Thus, it is critical that bed elevation measurement interval be as fine as possible and that there is awareness that censoring of the smallest particle residence times may occur. Second, selection of vertical discretization for identification of particle entrainment/disentainment may affect interpretation of residence time distributions. For example, entrainment of a particle that spans many measurement levels will be interpreted as multiple particle entrainments. On the other hand, deposition/entrainment may affect interpretation of residence times. Henceforth, we refer the unconditional

As described below, bed elevation fluctuations tend to be symmetric and the overall residence time density for a point on the bed tends to fall in the middle of the residence time densities for each level. This centering tendency indicates that measurement bias in residence time for the largest and smallest stones does not significantly affect the overall shape of the residence time density curve.

[12] Gaussian bed elevation distributions (Figure 2b) have been assumed [Yang and Sayre, 1971; Wong et al., 2007] or observed [Nikora et al., 1998] in gravel bed rivers, although they have been demonstrated through extensive laboratory flume experiments to deviate from Gaussian with increased bedform development [Marion et al., 2003]. The collection of residence times collected from a time series of bed elevations at all elevation levels (Figure 2c) from flume or field studies then defines the unconditional sediment residence time distribution (Figure 2d). The unconditional empirical distribution of residence times $P(R > r)$, where $P(*)$ denotes probability, can be computed from the conditional residence time distribution for each bed elevation, $P(R > r | z_k)$ (Figure 2d), by creating a distribution where probability at each level is weighted by the likelihood of deposition at each of those levels through the bed elevation density, $h(z_k)$ [Nakagawa and Tsujimoto, 1980]. Then the unconditional residence time distribution is given by (similar to Yang and Sayre [1971])

$$P(R > r) = \int P(R > r | z) h(z) dz$$  \hspace{1cm} (1)
residence time distribution given by equation (1) as an overall residence time distribution. Over a time period, the probability that the particle will be reentrained following an earlier deposition is equal to the probability that the bed surface will return to and cross the deposition elevation at least once during that time period.

Our goal is to link the statistics of bed elevation fluctuations with sediment residence time. We begin with the assumption that at the scale of experimental measurements, bed fluctuations have a large random component. Specifically, we do not have the capability of predicting subsequent bed elevations with a degree of certainty beyond that provided by statistics of previous elevations. We start by defining a bed elevation series on a continuum. Let \( z_\eta \) denote continuous bed elevation. Then

\[
\{ z_\eta(t) : 0 \leq t \leq T \}
\]

is the continuous time series of bed elevations \( z_\eta \) up to time \( T \). At low flow rates, it is unlikely that extreme bed elevations \( z_\eta \) in the form of scour or fill will occur and so we assume that random vertical fluctuations have a well-defined mean \( E(z_\eta) = \mu_\eta \) and variance \( \text{var}(z_\eta) = \sigma^2_\eta \). We hypothesize that bed elevation equation (2) is well represented by a random walk model where (1) the magnitude of fluctuations is independent of one another, (2) fluctuations have some short-range negative dependence as scour and fill events alternate, or (3) fluctuations have long-range negative dependence that persists over a wide range of timescales. To clarify, negative dependence implies that bed elevation fluctuations will alternate between positive and negative as holes are preferentially filled and high points are preferentially eroded. These distinctions are significant because the asymptotic stochastic theory we use to model residence time distribution depends on these characteristics. In the long timescaling limit, an elevation fluctuation random walk with independent fluctuations or short-range correlated fluctuations will be well represented by a Brownian motion model. If negative correlation in fluctuations decays slowly enough, then the random walk will instead converge to a fractional Brownian motion with Hurst coefficient (correlation parameter) \( 0 < H < 0.5 \), so that here fractional Brownian motion is a continuous stochastic process in which negative correlation persists at all scales [Mandelbrot, 1982] and the persistence of that correlation decreases from zero to 0.5. Note that classical Brownian motion is a subset of fractional Brownian motion in which \( H = 0.5 \).

### 3. Theory

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### 3.1. Semi-Infinite Bed

For heuristic purposes, we first consider a theoretical semi-infinite bed where bed elevation at a point in the channel can fluctuate between zero and infinity (see simulation in Figure 3a, black curve). Even when the path that a surface following a Brownian motion with mean-zero increments can follow is unbounded above, there is a high likelihood of many short excursions from a given starting point. However, it is also possible for the bed elevation to travel extremely large distances from the deposition point or to fluctuate above the deposition point for very long periods. The result is that the residence time for particles deposited at a given elevation follows a heavy-tailed power law distribution (Figure 3b, black points). Specifically, it is well...
known that the exceedance distribution function for returns to a point for a Brownian motion path decays as $P(T > t) \sim t^{-1/2}$ and, more generally, the exceedance distribution of return times for a fractional Brownian motion decays as $P(T > t) \sim t^{-H-1}$, where $H$ is the Hurst coefficient described above [Ding and Yang, 1995]. It is worth mentioning that the power law decay of a distribution is not a function of the size of individual events but the decay in the probability of finding events extremely far from the majority of the population. The same power law decay can be observed in the distribution from a sample where the mode is 1, 100, or 1,000,000. The return time to every level of a (fractional) Brownian motion decays as $P(T > t) \sim t^{-H-1}$, whether the level is near the origin or far from it.

3.2. Finite Bed

[17] We have established that the residence time for particles at any level will follow a power law when possible bed elevations are unbounded above. However, fluvial channels have a finite bed elevation with bed elevation-driven physical conditions of sediment load and caliber, flow regime, and hydraulic drag on the bed surface [Knighton, 1998]. Fluvial channels are also bounded below by the boundary between readily mobile and immobile sediment layers, which in a bed of coarse material (i.e., primarily gravels) in natural channels seldom exceed 2D$_{90}$ grains [Haschenburger and Church, 1998; Wilcock et al., 1996]. The existence of an upper bed elevation $L$ means that a conceptual model should not allow for unbounded, extreme changes in upward bed motion. Instead, the conceptual model for this case is that bed elevation follows random walk that starts at $0 \leq z(t_0) \leq L$ and can fluctuate in the interval $[z(t), L]$. This can only start when a particle is deposited at $z(t)$. The bed is reflected downward any time it hits $L$, and we can think about $z(t)$ as an absorbing boundary as, again, the residence time for a particle at $z(t)$ is equivalent to the time until the return of the bed from above to that elevation (Figures 3a and 3b). This stochastic model is known as the reflection mode for the first passage on a bounded interval [Redner, 2007, Chapter 2] and has been applied to the wet period of a finite dam, or busy period of a finite queue [Kinateder and Lee, 2000].

[18] Under this stochastic conceptual model, the residence time distribution will be a function of how close $z(t)$ is to $L$. If $z(t)$ is far from $L$, residence time may be close to the pure power law for the infinite bed case, where there was a high likelihood of extremely long residence times. However, as $z(t)$ gets closer to $L$, there is an increased likelihood that the series will encounter the reflective boundary much sooner resulting in higher likelihood of shorter residence times. In the extreme case, where $z(t) = L$, sediment residence time will be zero. First, we will characterize the dependence of sediment residence time on the relationship between particle depth $z(t)$ and the upper bound on bed elevation $L$. The residence time distribution is typically represented by its Laplace transform and does not have a solution in closed space [Kinateder and Lee, 2000].

[19] Since we generally do not measure bed elevation series in the laboratory or field on a continuum, measurements are instead made at discrete time intervals $\Delta t$ where we define the discrete subset of the continuous time series (equation (2)) indexed over $t \in \{0, 1, \ldots, T/\Delta t\}$. The crossover time $T_e$ between power law and exponential character [Redner, 2007],

$$T_e = \frac{(L - z_0)^2}{\sigma^2/\Delta t},$$

is a function of deposition depth below the bed surface $(L - z_0)$ and the rate at which the bed fluctuates around its mean elevation (i.e., the dispersion coefficient $D = \sigma^2/\Delta t$ of the random walk), where $\sigma^2$ is a constant variance of the bed fluctuations for all $t$. Well below this cutoff, the residence time distribution will decay as a power law (the probability $P(T > t) \sim t^{-H-1}$ will decay as density $p(t) \sim t^{-H-2}$). Well above $T_e$, the residence time distribution will decay exponentially with exponent [Redner, 2007]

$$\beta = \frac{\sigma^2/\Delta t}{z_0(L - z_0)}.$$  

[20] Finally, very close to the crossover time, there will be an enhancement in the return probability that arises from the overlap in residence times that have a significant probability of occurring from either the power law returns or the reflected exponential returns. This enhancement exists in the residence time data of Figure 1. Although the residence time distribution cannot be written exactly in closed form, the tempered Pareto distribution provides a useful approximation [Redner, 2007], as it captures the asymptotic behavior of an infinite system and the finite-size cutoff imposed by the maximum bed elevation. However, because of the enhancement in residence time probabilities that occurs around the crossover time in data, it may be difficult to use the empirical residence time distribution to measure exactly the power law or exponential exponents for the appropriate tempered Pareto distribution.

[21] Now that we have specified the residence time distribution for particles at specified level $z_0$, what does the overall residence time distribution that could be used in a sediment transport model look like? Like distributions for the individual particle depths, the overall distribution will be well approximated by a tempered Pareto distribution with a transition time that is a weighted average of all the levels and will look most like the distribution around the mode bed elevation. If the transition time from power law to exponential residence time behavior is smaller than the timescale of interest for transport studies, an exponential residence time will suffice. If the transition time is similar to or larger than the study period of interest, then a full tempered Pareto residence time distribution will be required to capture both the power law and exponential portions of the distribution. Again, the power law portion of the residence time distribution will contain information about the independence or long-range correlation in the bed fluctuation data and can be estimated from observations.

[22] The conclusions of our theoretical analysis are as follows:

[23] 1. Under conditions of no aggradation or degradation, bed elevation will fluctuate around an average value. The most deeply buried particles will have residence time distributions that start as power law but cross over to exponential at crossover time $T_e$. The crossover time from power law to exponential residence time distribution for all particles will be a weighted average over all depths.

[24] 2. The power law governing early residence time will be related to the presence of long-range dependence in the
If long-range dependence does not exist, then the power law exponent will be approximately 0.5. Significant long-range dependence will result in a power law with a smaller exponent.

### 4. Case Studies

We use two elevation time series (Figures 4a and 4e) from flume experiments performed at the St. Anthony Falls Laboratory. Quantile-Quantile (Q-Q) plots and histograms demonstrate that the “simple case” with well-sorted grains and no bedforms has a nearly Gaussian bed elevation (Figure 4b) and bed fluctuation distributions [Wong et al., 2007] while the “complicated case” with poorly sorted grains and large-scale bedforms follows a non-Gaussian bed elevation distribution (Figure 4f) with symmetric fluctuations [Singh et al., 2009]. A straight line in the Q-Q plots indicates that the sample distribution coincides with a theoretical Gaussian distribution showing the former has the same distribution as the latter. Both flume experiments were conducted under equilibrium conditions such that changes in bed surface reflect fluctuations about mean bed elevation. For each experiment, the entrainment probabilities (Figure 4c and 4g) for particles deposited at any level were computed by integrating the bed elevation histograms. These experiments represent the simplest, ideal conditions so that variations in bed elevation are not influenced by variations in flow or sediment feed but rather particle and channel characteristics. Here we chose two cases from a number of flume experiments based on their differences in grain size sorting, bedform development and bed elevation distributions. The first flume experiment was conducted in a 0.5 m wide, 0.9 m deep, 27.5 m long straight flume using well-sorted sediment on planar bed morphology under a low flow conditions [Wong et al., 2007]. This experiment was conducted in the last 22.5 m length of the flume. The second flume experiment was conducted in a 2.74 m wide, 1.8 m deep, 55 m long straight flume using poorly sorted sediment under high flow.

**Figure 4.** Sediment residence time distributions were estimated from flume experiment bed elevation data for no bedform morphology and bedform cases. (a, e) Time series of bed elevations with (b, f) a histogram of elevations along with Q-Q plot inset showing departure from Gaussian, (c, g) entrainment probabilities, and (d, h) overall residence time distribution (black dots) fitted to exponentially tempered Pareto distributions (black lines) and mean crossover times (vertical black dashed lines). The overall residence time distribution is the union of all sets of elevation specific residence time distributions (red lines).
conditions that resulted in large-scale bedform morphology [Singh et al., 2009]. This flume experiment was conducted in the last 20 m length of the flume. The higher flow conditions resulted in large-scale bedform development. Both experiments used five sonar transducers to record fluctuations in bed elevation at intervals of 3 s for the no bedform case and 10 s for the bedform case. Sampling rates are critical in estimating residence time from a bed elevation series since low sampling frequency may result in missed fluctuations in bed elevation. Thus, it is always better to err on the side of too high rather than too low sampling frequency. However, we might expect differences in resulting residence time distributions to be negligible if only a few elevation fluctuations were missed. Experimental conditions and bed elevation statistics for both flume experiments are shown in Table 1. Elevation fluctuation correlations were estimated for each elevation series as 0.48 and 0.74 for the simple and complicated cases, respectively, using a corrected empirical Hurst exponent [Anis and Lloyd, 1976]. Correlograms for each elevation time series show no long-range correlation (Figure 5).

26 Residence time for particles deposited at each elevation within the bed and also the overall residence time distributions were calculated from bed elevation data for both the no bedform case (Figure 4d) and the bedform case (Figure 4h). An exponentially tempered Pareto distribution (black curves),

\[ P(T > t) = \lambda t^{-\alpha} \exp(-\beta t), \quad t \geq t_0 > 0, \quad (5) \]

was fitted to each of the overall residence time distributions where fitting parameters are \((\alpha, \beta, \gamma)\) and \(\lambda = \gamma t_0 \exp(\beta t_0)\) [Meerschaert et al., 2012]. Overall residence time distributions were computed by cumulating all residence times for each elevation level (Figures 4d and 4h, red curves) into a single set. The crossover time from power law to exponential residence time (Figures 4d and 4h, vertical dashed black line) for the overall residence time was estimated by first calculating crossover times for each elevation using equation (3) using \(L = \max \{z\}\) and then calculating the mean crossover time by weighting

\[ T_c = \frac{\sum_{i=1}^{n} f(z_i)(T_i)}{\sum_{i=1}^{n} f(z_i)}, \quad (6) \]

where \(f(z_i)\) is frequency of bed elevation measurements at elevation \(z_i\) and \((T_i)\) is its corresponding crossover time. Crossover times for the flume experiments were 19 and 94 s for the no bedforms and bedforms cases, respectively. This method can be extended to the entire bed surface using a lidar bed elevation series and assuming the bed surface as a random field, \(Z(x, y, t)\), where \(x\) and \(y\) are point locations in the horizontal plane. The two examples used herein would have notation \(z(t) = Z(x_0, y_0, t)\) for fixed location \((x_0, y_0)\).

Figure 5. Correlograms of autocorrelation functions for (a) simple case of well-sorted grains with no bedforms, and (b) complicated case of poorly sorted grains with large-scale bedforms. Actual time lags rather than lag steps were used in the horizontal axis to facilitate series comparison.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Simple Case</th>
<th>Complicated Case</th>
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<tr>
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<tr>
<td>discharge</td>
<td>m³ s⁻¹</td>
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<tr>
<td>sediment rate</td>
<td>kg s⁻¹</td>
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<tr>
<td>bed slope</td>
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<td>1.05</td>
<td>n/a</td>
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<tr>
<td>water slope</td>
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<td>1.03</td>
<td>0.53</td>
</tr>
<tr>
<td>water depth</td>
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<td>1.8</td>
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<tr>
<td>Shields stress</td>
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<td>–</td>
<td>0.105</td>
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<td>Sediment statistics</td>
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<tr>
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<td>8.5</td>
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<td>90%tile grain size</td>
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Simple case is well-sorted sediment with plane bedform morphology. Complicated case is poorly sorted sediment with large scale bedforms. Although each experiment was performed under equilibrium conditions, each was chosen for their differences in grain size sorting, bedform development, and Gaussian (simple) versus non-Gaussian (complicated) elevation distributions.
5. Discussion

[27] The stochastic conceptual model for sediment residence time as a return time of the bed was effectively developed by the 1970s [Yang and Sayre, 1971; Nakagawa and Tsujimoto, 1980]. The reanalysis in this study benefited from advances in stochastic modeling and measurement techniques in the 40 years since. While there has been debate regarding the theoretically best distribution to choose for sediment residence time distributions, an empirical density can now be measured explicitly at any point on a bed for individual or cumulative depths. Even in a non-aggrading bed under steady state flow and sediment load, the residence time of newly emplaced tracers will be transient, as the tracers “sample” the variety of waiting times possible in the bed. This transient period exhibits power law residence times as laboratory data confirm that the return time of the bed to a given elevation will have power law character far from the upper bound of the active layer for a finite period and then transition to exponential character. This happened in 19 and 43 s in the laboratory experiments described above, but these transition times are rapid since flume experiments are performed under simple conditions. In field cases, more complicated fluvial systems with wide-ranging sediment supply and flow conditions as well as diverse morphology and sediment history lead to more stable bedforms and hence much longer crossover times than would occur in flume experiments [Knighton, 1998]. Where there are no bed elevation measurements, an analysis of mobile tracer mass decline can be used to estimate the crossover time. Specifically, if a collection of tracers is seeded on the surface of a channel bed, as gravel begins sampling a wide distribution of residence times in the bed, there will be a net accumulation of tracers in the bed and so the fraction of tracers that remain mobile declines as a power law. The crossover time to exponential-type residence times occurs after the longest residence time has been sampled, mobile-immobile exchange is balanced, and the mobile fraction stabilizes. For example, transition time for the gravel bed channels of Duck Creek and Bolmoral Canal was estimated (via computations of tracer mass stability) to be approximately 500 s, while the same quantity for the sand-bed channel of North Loup River was approximately 100 h [Zhang et al., 2012]. Residence times in Duck Creek were observed to be a result of insufficient bed shear stresses rather than burial by migrating bedforms or bedload sheets [Drake et al., 1988]. While Nikora et al. [2002] describe a global long-time regime that will be subdiffusive because of power law rest periods, this study suggests that subdiffusive transport will instead be part of an intermediate regime that will eventually convert to a diffusive transport regime after the longest resting times have been incorporated into the overall residence time distribution.

[28] We considered bed elevation and residence time in flumes under two experimental conditions: one performed with well-sorted sediment on plane-bed morphology and the other performed with poorly sorted sediment on large-scale bedform morphology. The distribution describing likelihood of bed elevation for each level was symmetric and near-Gaussian in one case and non-Gaussian in the other. Despite these differences, residence time distributions follow similar statistics in the two cases. Although physical processes in the flume affect bed elevation, the residence time distributions for both experiments are related to the statistics of first passage processes on bounded intervals rather than physical characteristics of the channel or particles. Since both experiments were performed under equilibrium conditions, how might we expect residence time distributions and crossover times to change under varying conditions of sediment supply and channel scour? High sediment supply is likely to result in little to no armoring on the bed where we might expect more fluctuation of the bed surface resulting in decreased residence and crossover times. Low sediment supply is likely to lead to channel armoring where we might expect vertical adjustment of the bed to occur less frequently resulting in increased residence and crossover times. Scour leads to thicker active layers where particles are buried deeper in the channel. Here we might expect increased residence and crossover times where the timing of crossover and extent of sediment residence largely depends on scour frequency and depth.

[29] In natural streams, sediment transport conditions are more complicated than flume experiments since sediment supply and flow conditions vary over long timescales, and changes in climate, geology, tectonics, vegetation, and land use all work to influence channel morphology. Differential movement of bed material over time and space creates large channel features, such as bars, riffles, pools, and small structures such as clusters and other microforms. Residence time of sediment in the channel has been observed to differ over morphological features. Bars have a preferential burial signature with the thickest active layers, which provide long-term sediment storage, being observed to have the longest residence times. Conversely, the channel thalweg is likely to have an armored surface with the thinnest active layer, which resides in the fastest water, being observed to have the shortest residence times. Residence times of sediment in riffle-pools are a function of flow conditions since riffle sections scour during low flow and fill during high flow while pool sections fill during low flow and scour during high flow. Shorter residence times for sediment close to the surface can readily mobilize during a flood, whereas those buried deeper in the substrate remain immobile over longer periods. This behavior is realized by observing increased residence times with increased depth in the bed elevation series (see Figure 2c). Deeper scour increases sediment residence time, which in turn decreases virtual velocity of streamwise travel distance of sediment in the channel. In bedform transport, sediment residence times can be correlated with length of motion [Shen and Cheong, 1980] and a more sophisticated stochastic model which includes the relationship between step length and residence time may be needed to reproduce overall transport characteristics. Several tracer surveys in the field, where flood magnitudes were slightly higher than critical values sufficient for localized scour and fill, were found to have heavy-tailed travel distances between surveys [Hassan et al., 2013]. Sediment in areas of the channel unaffected by scour was not entrained during the event and hence continue to lengthen residence time in the bed.

[30] Some bedload models assume a constant active layer thickness [Hirano, 1971] often assumed to be 2D90; however, we know that scour depth is highly variable. Bed elevation measurements at a point and across a channel should be used to infer both representative active layer thickness and variations in it over the channel for bedload models. In the no bedform experiment described above, active layer depth based on 2D90 particle size is 19.2 mm, corresponding with the
majority of the bed elevation histogram (Figure 4b). In the bedform case, active layer depth based on \(2D_{90}\) particle size is 53.6 mm, only a fraction of the mobile bed depth (Figure 4f). The discrepancy in these results is attributed to the two primary differences in experimental conditions. The lower flow in the no bedforms case does not produce bedforms resulting in a thinner active layer, whereas the higher flow in the bedforms case produces bedforms resulting in a thicker active layer. Particle size distribution also contributes to active layer thickness with the smaller, uniform grain size in the no bedform case resulting in thinner active layers and larger, poorly sorted grain size in the bedforms case resulting in thicker active layers. A statistical measure of the active layer from bed elevation data may be useful in the future. On the other hand, many contemporary bedload models are based on sediment continuity rather than active layers [Parker et al., 2000; Blom and Parker, 2004]. Use of bed elevations measurements to calculate residence time at each level in the bed fits nicely into the sediment continuity framework.

[31] Here we quantified overall residence time distribution and mean crossover time for sediment from bed elevation series obtained from flume experiments where fluctuations in bed elevations were measured with sonar transducers at high-frequency sampling rates. These were simple flume experiments, and their residence time distributions were quantified for bed elevation at a single point. Improvements in surface measurement methods used in the field allow for high-resolution bed elevation measurements across the channel. Lidar is now capable of submillimeter-scale resolution [Piracha et al., 2010], which is sufficient to capture individual coarse grain sizes at the channel surface as well as subtle elevation changes on the bed. This facilitates development of overall residence time distributions for sediment over a reach provided that sampling rates are sufficient to capture fluctuations in bed elevation. By considering bed elevation as a random field, \(Z(x,y,t)\), where \(x\) and \(y\) are point locations in the horizontal plane, we can now calculate residence times over a reach with high precision. Such high resolution is sensitive enough to estimate residence times during low flow conditions when scour and fill are localized and vertical mixing is minimal. Bed elevation fluctuation measurements taken over a river reach will compliment tracer studies, allowing quantification of sediment dispersion and residence time distributions influence by the same flow and sediment supply conditions. This will facilitate future study into their duality so that we may further characterize particle exchange between mobile and immobile zones in order to improve bedload transport models. Finally, stronger links between hydraulics, sediment distributions, and bed topography—combined with this work—will also inform the relationship between physical setting and sediment residence time distribution.

6. Conclusions

[32] 1. Residence time and entrainment probability for all levels in a streambed are captured in lidar or sonar transducer-recorded bed elevation measurements. Limitations exist with respect to spatial and temporal measurement intervals.

[33] 2. Sediment residence time at a given depth can be conceptualized as a stochastic return time process on a finite interval. Overall sediment residence time is an average of residence times at all depths weighted by the likelihood of deposition at each depth.

[34] 3. When tracers are seeded on the bed surface, power law residence time will be observed until a timescale set by the bed thickness and bed fluctuation statistics \(T_e = \frac{(\zeta_{0}/\sigma^2)}{t}\).

After this time, the long-time (global) residence time distribution will take exponential form. The crossover time in flume studies can be on the order of seconds to minutes, while the few analyses that have been performed in natural channels suggest that the crossover time in rivers can be days to years.

[35] 4. The tempered Pareto model is a useful approximation for residence time distributions in analytical models of transport as suggested by Zhang et al. [2012]. Whether a model needs to incorporate residence times that are strictly power law, transition from power law to exponential, or exponential, depends on the timescale of interest.

36 Acknowledgments. We are grateful to the National Center for Earth-surface Dynamics (NCED) for easily accessible laboratory data. Arvind Singh helped us access and understand the data. H. Voepel was supported by a Desert Research Institute George Burke Maxey Fellowship and a CUAHSI Pathfinder Fellowship. R. Schumer was partially supported by the Science and Technology Center Program of the National Science Foundation via NCED under agreement EAR-0120914.

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